

# On the $r$ -Equitable Coloring of Graphs

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## Abstract

A graph  $G$  consists of a nonempty vertex set  $V(G)$  and an edge set  $E(G)$ . All graphs considered in this talk are finite, loopless, and without multiple edges. Let  $k \geq 1$  be an integer. A (proper)  $k$ -coloring of a graph  $G$  is a mapping  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  such that adjacent vertices have different images. The images are called *colors* and all vertices of a fixed color constitute a *color class*. Then a  $k$ -coloring of a graph  $G$  is said to be *equitable* if the sizes of any two color classes differ by at most one. And a graph  $G$  is *equitably  $k$ -colorable* if  $G$  has an equitable  $k$ -coloring. So far, quite a few results on the equitable coloring of graphs have been obtained.

Recently, Hertz and Ries [1] generalize the notion of equitable colorability. They say that a  $k$ -coloring of a graph  $G$  is  *$r$ -equitable* if the sizes of any two color classes differ by at most  $r$ , where  $r \geq 0$  is an integer. And a graph  $G$  is  *$r$ -equitably  $k$ -colorable* if there exists an  $r$ -equitable  $k$ -coloring of  $G$ . Clearly, an equitably  $k$ -colorable graph is 1-equitably  $k$ -colorable, and vice versa. Besides, the least  $k$  such that a graph  $G$  is  $r$ -equitably  $k$ -colorable is called the  *$r$ -equitable chromatic number* of  $G$  and denoted  $\chi_{r=}(G)$ . Also, the least  $n$  such that a graph  $G$  is  $r$ -equitably  $k$ -colorable for all  $k \geq n$  is called the  *$r$ -equitable chromatic threshold* of  $G$  and denoted  $\chi_{r=}^*(G)$ .

In this talk, we will give a brief survey of some recent progress on the  $r$ -equitable coloring of graphs, especially for  $r \neq 1$ .

**Keywords:** Equitable coloring;  $r$ -Equitable coloring;  $r$ -Equitable chromatic number;  $r$ -Equitable chromatic threshold.

## References

- [1] A. Hertz, B. Ries, On  $r$ -equitable colorings of trees and forests, submitted, 2011. (<http://www.gerad.ca/alainh/Ries.pdf>)

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